ASYMMETRIC MULTI-PHASE DEFORMABLE MODEL FOR COLON SEGMENTATION

Yong Seok Yoo†, Kyoung Mu Lee†, Il Dong Yun‡, and Sang Uk Lee†

†Seoul National University, ‡Hankuk University of Foreign Studies

ABSTRACT

In virtual colonography, precise segmentation is essential for accurate diagnosis. For the segmentation of colon wall, we propose a novel multi-phase deformable model using a level set method. By defining an asymmetric energy functional, the proposed model can simultaneously segment regions with different characteristics. Compared with the conventional multi-phase models, it shows better convergence without ambiguity. Experimental results with real CT images demonstrate that the proposed algorithm outperforms other methods.

1. INTRODUCTION

Recently, medical images such as CT or MRI are widely used for medical diagnosis. One of the promising applications of medical image is virtual colonography [1]. A virtual 3D colon model is reconstructed from abdominal CT images, and can be used for the detection of colorectal polyps and cancers. Physicians can scrutinize the colon walls without inserting an optical endoscope into patients. Thus, the virtual colonography reduces the time, cost, and pain compared to traditional optical colonography.

For accurate reconstruction of the colon model, precise segmentation of it is essential. However, in general, the segmentation of colon is not an easy task. Although, the air in colon can be easily segmented by simple thresholding or region growing methods, other remaining materials such as fluid or stools make it difficult to segment the true boundary of colon. So far, not much works has been done on this problem. In this paper, we propose a new colon segmentation algorithm based on deformable model using level set.

Deformable models are known to handle complex-shaped objects, and the prior knowledge can be easily combined to the model. Kass *et al.* first proposed a deformable model called snake [2]. Although snake can represent an arbitrary shape, it cannot handle the topological changes well. Topological flexibility is fulfilled by using the level set method [3]. Using multiple level set functions, multi-phase deformable models can segment multiple objects simultaneously [4, 5, 6]. There are largely two kinds of approaches for multi-phase models. One is to associate each region with a level set function and add some constraints [4, 5]. The other is to represent regions by the combination of several level set functions [6].

The first approach is proposed by Zhao *et al.* [4]. They proposed a variational level set approach to deal with multiple phases with the same number of level set functions. Using this framework, Samson *et al.* considered segmentation as the optimal partition problem [5]. This approach is a kind of constrained minimization problem, which is solved by the Lagrange multiplier. However, the computational complexity of the Lagrange multiplier is enormous. Moreover, it has serious problems of vacuum or overlap.

The other approach is proposed by Vese and Chan [6]. By using vector level set functions and vector Heviside functions, they represented regions by the combination of the fewer number of level set functions.

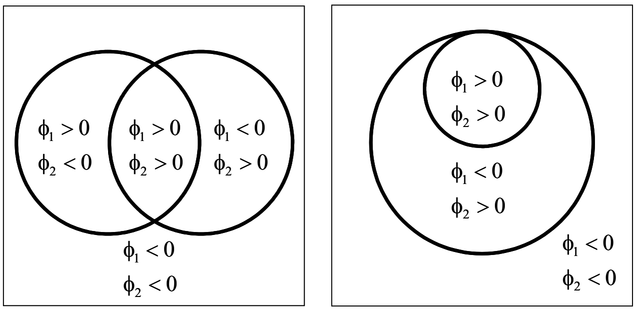
The second approach has several advantages. First, it uses fewer number of level set functions compared to the multi-phase model by constraints [4, 5]. Second, it does not need additional constraints. Thus, the complex step of calculating Lagrange multiplier can be avoided. Third, there is no vacuum or overlap region problems [6].

However, the multi-phase model by combination has still limitations. First, it shows poor convergence. Because the energy functional is not convex, the model is sensitive to the initialization and often falls into local minima. Second, it has ambiguity. Because there is no connection between level set functions, one cannot predict which level set functions will converge to which region. Third, the model performs poorly in real images when the intensity difference is not distinct.

In order to solve these problems, a novel asymmetric multi-phase deformable model using a level set method is proposed. The proposed model has a simpler energy functional and shows better convergence property. It can deal with the regions with different characteristics simultaneously and fits for real CT images.

2. PROPOSED ASYMMETRIC MULTI-PHASE DEFORMABLE MODEL

The deformable model by Vese and Chan has no preference to different regions, and has a symmetric energy functional with respect to each level set function. Thus, let's call this



(a) Symmetric model (b) Asymmetric model

Fig. 1. Symmetric and asymmetric models.

model *symmetric*. n regions are represented by $m = \log_2 n$ number of level set functions. A vector level set function is defined by $\Phi = (\phi_1, \phi_2, \ldots, \phi_m)$, where each ϕ_i is individual level set function. Likewise, a vector Heviside function is defined by $H(\Phi) = (H(\phi_1), H(\phi_2), \ldots, H(\phi_m))$. Because the value of each Heviside function has either 0 or 1, the vector Heviside function has $n = 2^m$ cases. For example, in 4-phase deformable model four regions are represented by the two level set functions ϕ_1 and ϕ_2 as shown in Fig. 1(a).

Contrary to the symmetric model, the proposed asymmetric model segments the image into two overlapping regions and background using the same number of level set functions as shown in Fig. 1(b). The inner region is defined as the region where the values of all level set functions are positive. In addition to the inner region, the outer region covers the region where one level set function, in this case ϕ_1, is negative. The background is defined by the area where the value of all level set functions are negative.

The asymmetric deformable model is calculated by minimizing an asymmetric energy functional:

$$F = \int (u_o(x,y) - c_1)^2 \chi_{11} + \int (u_o(x,y) - c_2)^2 \chi_{01} + v \int |\nabla H_2(\phi_2)|, \quad (1)$$

where c_1 is the average intensity of the inner region represented by ϕ_1, and c_2 is that of outer region represented by ϕ_2. χ_{11} and χ_{01} are defined by

$$\chi_{11}(x,y) = \begin{cases} 1, & \text{if } \phi_1 > 0, \phi_2 > 0 \\ 0, & \text{otherwise}, \end{cases} \quad (2)$$

$$\chi_{01}(x,y) = \begin{cases} 1, & \text{if } \phi_1 < 0, \phi_2 > 0 \\ 0, & \text{otherwise}. \end{cases} \quad (3)$$

The asymmetric model has a simpler energy functional compared to the symmetric model. Each term in the energy functional has following physical meanings; minimization of the first term forces both the inner region and the outer region to contain the area whose intensity is about c_1. Minimization of the second term forces only outer region to contain the area whose intensity is about c_2. The final term is the regularization term. As a result, the outer region represented by ϕ_2 includes the inner region represented by ϕ_1 without additional constraint.

Using the Heviside function H, the asymmetric energy functional is expressed by

$$F = \int (u_o(x,y) - c_1)^2 H(\phi_1) H(\phi_2) + \int (u_o(x,y) - c_2)^2 (1 - H(\phi_1)) H(\phi_2) + v \int |\nabla H(\phi_2)|. \quad (4)$$

Equation (4) can be minimized by the following Euler-Lagrange equations:

$$\delta(\phi_1)[((u_0 - c_1)^2 - (u_0 - c_2)^2) H(\phi_2)] = 0, \quad (5)$$

$$\delta(\phi_2)[\nu \delta(\phi_2) div \frac{\nabla \phi_2}{|\nabla \phi_2|} - \{(u_0 - c_1)^2 H(\phi_1) + (u_0 - c_2)^2 (1 - H(\phi_1))\}] = 0. \quad (6)$$

Partial differential equations deduced from (5) and (6) are

$$\frac{\partial \phi_1}{\partial t} = \delta(\phi_1)[((u_0 - c_1)^2 - (u_0 - c_2)^2) H(\phi_2)], \quad (7)$$

$$\frac{\partial \phi_2}{\partial t} = \delta(\phi_2)[\nu \delta(\phi_2) div \frac{\nabla \phi_2}{|\nabla \phi_2|} - \{(u_0 - c_1)^2 H(\phi_1) + (u_0 - c_2)^2 (1 - H(\phi_1))\}]. \quad (8)$$

However, note that the boundary of the colon is usually blurred by the partial volume effect and the motion of the patient. It means that the intensity near the boundary is different from its average intensity. So, if we deform a model only by the intensity, the model may not capture the true boundary. Thus, in order to avoid this problem, we modify the partial differential equations by adding the gradient information to (7) and (8), and finally obtain

$$\frac{\partial \phi_1}{\partial t} = \delta(\phi_1) g(I)[((u_0 - c_1)^2 - (u_0 - c_2)^2) H(\phi_2)] + \overrightarrow{V} \cdot \nabla \phi_1, \quad (9)$$

$$\frac{\partial \phi_2}{\partial t} = \delta(\phi_2) g(I)[\nu \delta(\phi_2) div \frac{\nabla \phi_2}{|\nabla \phi_2|} - \{(u_0 - c_1)^2 H(\phi_1) + (u_0 - c_2)^2 (1 - H(\phi_1)\}] + \overrightarrow{V} \cdot \nabla \phi_2, \quad (10)$$

where $g(I)$ is an edge detector which is non-negative and has smaller value near the edges, and $\overrightarrow{V}$ is a gradient vector.

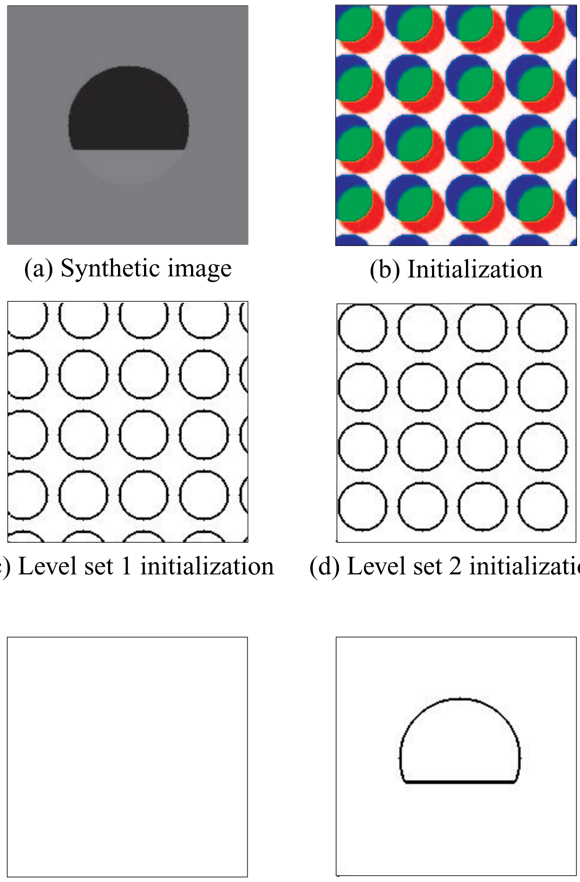
Calculation of the partial differential equations consists of two steps. First, ϕ_1 is determined iteratively by (9) keeping ϕ_2 fixed. And then once ϕ_1 is converged, ϕ_2 is calculated by (10) keeping ϕ_1 fixed. These two steps are repeated until both ϕ_1 and ϕ_2 converge.

3. SYMMETRIC VS. ASYMMETRIC

3.1. Symmetric Model

We have applied a symmetric 4-phase deformable model to a synthetic image as in Fig. 2(a). The image is a simplified model whose intensities are sampled from real CT images. Circle represents the inner wall of a colon. The upper dark area in the circle represents air in the colon. The lower area, whose intensity is slightly brighter than soft-tissue, represents other remaining materials. Background represents soft-tissue.

First, the multi-phase model is initialized as shown in Fig. 2(b). Initializing with a large number of overlapping small regions prevents the model from falling into local minima. Each color in Fig. 2(b) represents each region, and the two level set functions ϕ_1 and ϕ_2 are initialized as shown in Fig. 2(c) and (d), respectively. By the symmetric model, the boundaries of both level set functions have converged as shown in Fig. 2(e) and (f), respectively. One level set ϕ_1 was flattened away, and the other level set ϕ_2 converged to the dark region representing the air. So, the symmetric model failed to segment the lower region. In this case, only one level set function converges the region where the intensity difference is obvious. However, in general, it is ambiguous which level set will converge to which region.



(a) Synthetic image (b) Initialization (c) Level set 1 initialization (d) Level set 2 initialization (e) Level set 1 result (f) Level set 2 result

Fig. 2. Experiment of the symmetric model on a synthetic image.

3.2. Asymmetric Model

Now, we apply the proposed asymmetric multi-phase deformable model to the same synthetic test image. Asymmetric model is initialized differently. Inner and outer regions are initialized to the same region as shown in Fig. 3(b), and two level set functions ϕ_1, ϕ_2 are initialized identically as shown in Fig. 3(c) and (d), respectively. By the asymmetric model, the inner level set function converges to upper part of the circle, the air region, as shown in Fig.3(e). The converged outer level set function includes the inner region and also covers the lower part of the circle, remaining material, as shown in Fig. 3(f). Therefore, we can conclude that the proposed asymmetric multi-phase deformable model can segment regions which have multiple property without ambiguity. After the inner level set function finds one part, the outer level set function finds the entire region including the inner region. Furthermore, the model can capture the regions where the intensity difference is not distinct.

4. APPLYING ASYMMETRIC MODEL TO THE COLON SEGMENTATION

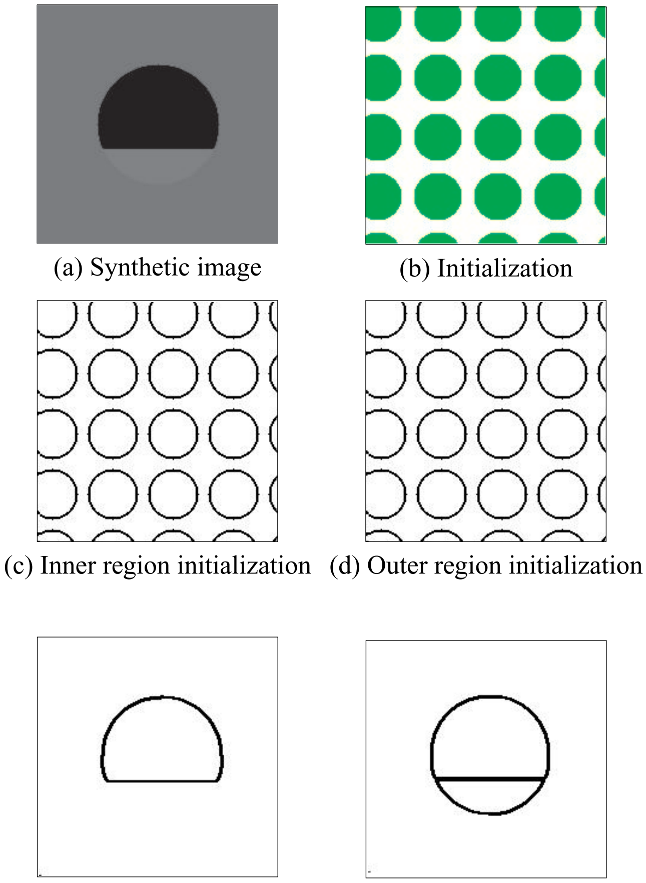
The asymmetric model shows good performance for colon segmentation. The model is initialized by the air in colon. After calculating intensity histogram of an image, the average intensity of each region c_i is estimated. First, all the air regions in an image is extracted by thresholding followed by the connected component analysis. For a CT slice image like Fig. 4(a), both of the inner and the outer regions are initialized by the air region as shown in Fig. 4(b). Now, level set functions ϕ_1 and ϕ_2 are constructed from the region by the fast marching method [3].

From the initial air region, the asymmetric model is deformed to the true colon boundary. While the inner region does not change as shown in Fig. 4(c), the outer region expanded to true colon boundary, including air and remaining liquid as shown in Fig. 4(d).

5. CONCLUSION

In this paper, we proposed a new asymmetric multi-phase deformable model which has the asymmetric energy functional suitable for colon segmentation. The proposed model segments a given image to three regions: inner region, outer region and background. Compared with the existing symmetric model, it shows better convergence without ambiguity. Because the model is implemented by two level set functions, it enjoys all the merits of the level set method. Experimental results show that the asymmetric deformable model-based method outperforms to other methods in real CT image segmentation.

This approach can be extended to 3D easily by using higher dimensional level set functions. Another further work will be applying the proposed model to segment other organ which is composed of multiple regions.

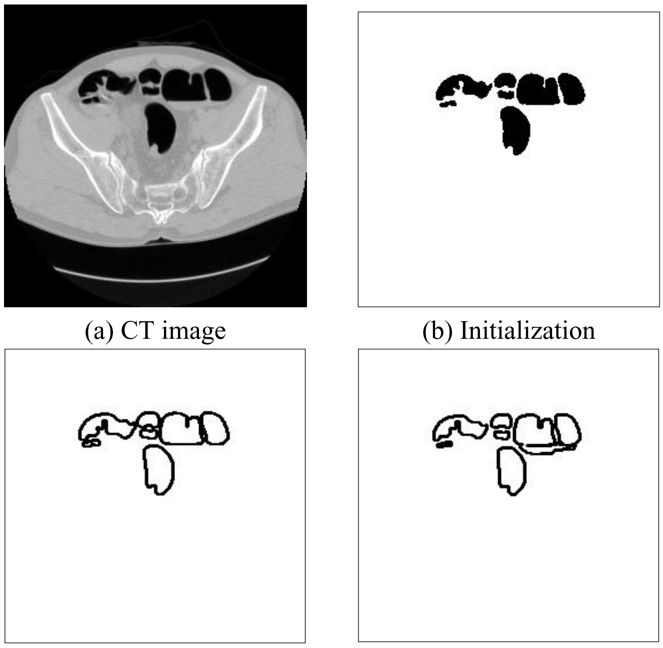


(a) Synthetic image (b) Initialization

(c) Inner region initialization (d) Outer region initialization

(e) Inner region result (f) Outer region result

Fig. 3. Experiment of the asymmetric model on a synthetic image.



(a) CT image (b) Initialization

(c) Inner region result (d) Outer region result

Fig. 4. Experimental results of the asymmetric model on a real CT image

6. REFERENCES

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